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Features of the linear intercept method used for measuring the grain size in WC–Co hardmetals

V. A. Pesin, M. V. Vasilyeva✉, **A. S. Osmakov**

“Virial” Ltd.
27 Bld. R Engels Prosp., Saint-Petersburg 194156, Russia

✉ VasilyevaMV@virial.ru

Abstract. Several WC–Co hardmetals with varying WC grain size distributions were analyzed to measure the mean grain size using the linear intercept (L) and planimetric (d_J) methods. Additional measurements included the equivalent diameter (d_{eq}) and mean chords (d_{ch}) for all grains, and separately, for grains intersected by the line. The findings show that mean sizes and size distributions of grains intersected by the line differ from those of all grains. This discrepancy is attributed to the linear intercept method’s rule for drawing secants, leading to “shadowing” where finer grains are obscured by coarser ones. The relationship between the mean sizes of all grains and those intersected by the line can be quantified using the “shadow” function S, which depends on the coefficient of variation (c_v) of the WC grain size distribution, as $d^a/d^l = 1 - S$. Experimental data illustrate that the mean equivalent diameter d_{eq} correlates with the linear intercept method L through equation $d_{eq}/L = 1.4(1 - S)$, and the relationship between the mean grain size d_J and L are described by the equation $d_J/L = 1.4(1 - S)\sqrt{1+c_v^2}$. The analysis of grain distributions by the equivalent diameters and mean chords showed that they equally describe the alloy grain size distribution. The length distribution of random chords obtained using the linear intercept method differs from the alloy grain size distribution due to the shadow effect, and also because the length distribution of random chords is always broader than the mean grain chord distribution. It is demonstrated that the length distribution of random chords is a convolution of the grain size distribution function and a function related to the grain shape.

Keywords: grain size, grain size distribution, linear intercept method, planimetric method, shadow effect

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Особенности метода секущих, используемого для определения размера зерна в сплавах WC–Co

В. А. Песин, М. В. Васильева✉, **А. С. Осмаков**

ООО «Вириал»
Россия, 194156, г. Санкт-Петербург, пр-т Энгельса, 27Р

✉ VasilyevaMV@virial.ru

Аннотация. На ряде сплавов WC–Co с различной шириной распределения зерен WC по размерам проведены измерения средних размеров зерен методом секущих (L) и планиметрическим методом ($d_{дж}$), а также эквивалентных диаметров ($d_{экв}$) и средних хорд ($d_х$) на всех зернах и отдельно на зернах, лежащих на секущих. Установлено, что как значения средних размеров, так и распределения по размерам зерен, лежащих на секущих, и всех зерен не совпадают. Это обусловлено правилом проведения секущих в методе секущих и связанным с ним «затенением» мелких зерен крупными. Показано, что отношение средних размеров всех зерен к средним размерам зерен на линиях можно описать с использованием «теневой» функции S, зависящей от коэффициента вариации (c_v) распределения зерен WC по размерам, в виде $d^в/d^л = 1 - S$. Экспериментальные соотношения между средним эквивалентным диаметром $d_{экв}$ и средним размером зерна по методу секущих L описываются выражением

$d_{экв}/L = 1{,}4(1 - S)$, а соотношения между средним размером зерна $d_{дж}$ и L – выражением $d_{дж}/L = 1{,}4(1 - S)\sqrt{1 + c_v^2}$. Анализ распределений зерен по величине эквивалентных диаметров и средних хорд показал, что они в одинаковой степени описывают распределение зерен сплава по размерам. Распределение случайных хорд по длине, получаемое в методе секущих, не соответствует распределению зерен сплава по размерам из-за теневого эффекта и из-за того, что распределение длин случайных хорд всегда шире распределения средних хорд зерен. Показано, что распределение длин случайных хорд является сверткой функции распределения зерен по размерам и функцией, связанной с формой зерен.

Ключевые слова: размер зерна, распределение зерен по размерам, метод секущих, планиметрический метод, теневой эффект

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Introduction

The core parameters of the microstructure of metals and alloys, which determine their mechanical and physical properties, are the grain size and material grain size distribution [1–6]. The grain size measuring methods use different characteristics of the grain to determine its size, which impedes the comparability of findings presented by different authors.

Historically, the linear intercept method [7] was the first one for estimating grain sizes in metals and alloys, including hardmetals. To this day, it is the most widely used technique despite the development of the image analysis methods. According to the linear intercept method, a series of parallel lines (secants) are drawn on the image of the alloy microstructure and the lengths of intercepts (random chords) intersecting each grain that happens to be on one of these lines are measured. It should be noted that the same grain cannot be intersected several times. The mean alloy grain size is taken to be the arithmetic mean of the lengths of these chords, hereinafter referred to as L. Some authors use the distribution of random chord lengths as the grain size distribution [4; 8–12].

The second most popular method for estimating grain sizes is the Jefferies planimetric method [13], in which the mean grain area is determined by dividing the cross-section area of the sample (image) by the number of grains contained in it. Then this area is converted into the diameter of a circle of the same area, dubbed in the literature as the equivalent circle diameter and referred to in this paper as d_J. For two-phase alloys, the cross-section area of the sample is recalculated in proportion to the volumetric content of the analyzed phase. The computer methods of image analysis enabled to measure various dimensional characteristics of individual grains, including their area. Therefore, the mean grain area is now determined by simply averaging the areas of individual grains. This method is used in various model calculations [14–16]. In a number of works, the circle equivalent diameter (d_{eq}) calculated based on the area of an individual grain or a mean chord (d_{ch}) [19] is taken as the size of an individual grain [6; 17; 18]. As the individual grain sizes are determined, their size distribution can be obtained, which provides more complete information about the grain composition of the material.

A number of papers compared grain sizes obtained by various methods. For spherical grains, the relationship between d_{eq} and d_{ch} was defined [9]. In [15], the values of L and d_J were independently determined by measurement for a number of WC–Co hardmetals. The d_J/L ratio varied from 1.10 to 1.40, the mean value being 1.15. In the same paper, as well as in [14; 16], the value of this ratio was determined on model structures. The crystals of various shapes were used to make arbitrary cross sections, on which random chords were drawn. Then the areas of these sections and the lengths of all chords were averaged, and the mean values were used to determine d_J, L and d_J/L. The resulting d_J/L values stood at 1.74 [15] and 1.35–1.75 [16].

These results differ significantly from the experimental data. The authors themselves [15] point out this discrepancy, but do not offer any explanation. A possible reason for this difference may be that the mean size of a random chord determined in the computer model for all grains may not coincide with the L value measured by the linear intercept method. According to the linear intercept method, the lines cannot intersect the grains twice (ISO 4499 2(2020)). To meet this requirement, in micrographs, coarse grains can “shadow” fine ones and the latter are not taken into account when the mean value is found. Therefore, the mean size of random chords determined for all grains may be smaller than the mean size of random chords on the lines, and the modeling will yield an overestimated value of the d_J/L ratio. This phenomenon of fine grains being “shadowed” will hereinafter be called a “shadow effect.” The shadow effect and its impact on the grain size measurements by the linear intercept method have not been addressed in the literature so far. As some researchers substitute the alloy grain size distribution for the distribution of random chords intercepted by secants, we will dwell on the relationship between these distributions.

The objective of this study is to measure the shadow effect; to determine whether the shadow effect depends on the nature of the WC grain size distributions; to assess the impact of the shadow effect on the mean WC grain size measured by the linear intercept method; to find correlation between the WC grain size distributions and the size distribution of random chords in the linear intercept method.

Measurement objects and methods

We compared the dimensional characteristics measured for all grains on the hardmetals cross sections and for the grains intersected by the line. The equivalent diameter d_{eq}, which is related to d_J, and the mean grain chord d_{ch} related to L were chosen as the dimensional characteristics of individual grains.

For the research, we selected 7 samples produced by Virial LTD (St. Petersburg), representing WC–Co hardmetals with 10 wt. % Co. The samples included alloys with narrow, wide and bimodal grain size distributions.

To identify the boundaries of the carbide phase grains, the samples were etched with Murakami solution. The microstructure of the samples was studied using a MIRA 3 scanning electron microscope (SEM) (Tescan, Czech Republic). We used the Fiji image processing software (USA) [20] and VideoTest – Structure 5.2 (Russia) to analyze SEM micrographs.

The built-in functions of these software packages enabled to measure the following grain dimensional characteristics:

1. Mean size of the intercept (random chord) L:

$$L = \frac{1}{n}\sum l_i,$$

where l_i is a random chord of the i^{th} grain intersected by the line.

Hereinafter, we will distinguish between two types of sizes: with the index “a” – the size taking into account all the grains in the micrograph; with the index “l” – the size taking into account only the grains intersected by the line.

2. Equivalent diameter $d_J^{\text{a,l}}$ (Jefferies method):

$$d_J^{\text{a,l}} = \sqrt{\frac{4}{\pi}\bar{A}},$$

where $\bar{A} = \sum \frac{A_i^{\text{a,l}}}{n}$, $A_i^{\text{a,l}}$ is the area of the i^{th} grain.

3. Mean equivalent diameter $d_{eq}^{\text{a,l}}$:

$$d_{eq}^{\text{a,l}} = \frac{1}{n}\sum d_{eq,i}^{\text{a,l}},$$

where $d_{eq,i}^{\text{a,l}} = \sqrt{\frac{4}{\pi}A_i^{\text{a,l}}}$ – is the equivalent diameter of an individual grain.

4. Mean chord $d_{ch}^{\text{a,l}}$:

$$d_{ch}^{\text{a,l}} = \frac{1}{n}\sum d_{ch,i}^{\text{a,l}},$$

where $d_{ch,i}^{\text{a,l}}$ (i stands for the grain number) is the mean grain chord determined as the mean value of all chords crossing the grain parallel to secants.

The total number of grains in micrographs for each alloy amounted to about 2000–3000 and the number of grains intersected by the line exceeded 1000. For each mean grain size, the standard deviation and variation coefficient (c_v) were determined.

Experimental results

Table 1 features the experimental results of measurements of the corresponding mean grain sizes with variation coefficients (c_v) of all the hardmetals under study.

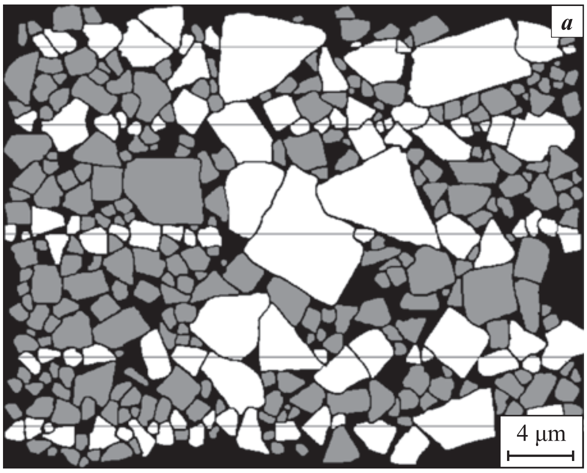
Table 1 shows that in all cases the mean sizes of the grains intersected by the line are larger than the mean sizes of all measured grains of a given hardmetal. This manifestation of the shadow effect is attributed to the reduced share of the fine fraction and, accordingly, the increased share of the coarse fraction in the grain size distribution. Fig. 1, *a* shows a fragment of one of the images processed in Fiji for sample *2* – the shadow effect is clearly seen there. Fig. 1, *b* demonstrates changes in grain distributions by the equivalent diameter for sample *3* with a relatively narrow distribution due to the shadow effect. Similar results were obtained for the mean chord of individual grains. Thus, when the mean grain size is determined by the linear intercept method observing the standard requirement, the changed values of initial mean sizes and the entire grain size distribution are obtained.

To measure the shadow effect, we considered the ratio of the mean sizes of all grains to the mean sizes of the grains intersected by the lines $d_{eq}^{\text{a}}/d_{eq}^{\text{l}}$ and $d_{ch}^{\text{a}}/d_{ch}^{\text{l}}$, as well as their dependence on the variation coefficients c_v. The experimental results are presented in Fig. 2. Since the monodisperse composi-

Table 1. **Dimensional parameters measured for the WC–Co hardmetals under study**

Таблица 1. **Измеренные размерные параметры для исследуемых твердых сплавов WC–Co**

Sample No.	d^{a}_{eq}/c_v	d^{l}_{eq}/c_v	L/c_v	d^{a}_{ch}/c_v	d^{l}_{ch}/c_v	d^{a}_{J}	d^{l}_{J}
	μm/rel. units					μm	
1	1.02/0.42	1.21/0.40	0.83/0.65	0.74/0.43	0.85/0.42	1.11	1.31
2	1.14/0.64	1.58/0.63	1.08/0.87	0.83/0.65	1.11/0.67	1.35	1.87
3	2.42/0.49	2.90/0.47	2.14/0.71	1.74/0.52	2.05/0.50	2.70	3.20
4	2.45/0.52	3.02/0.50	2.19/0.74	1.77/0.54	2.16/0.52	2.76	3.38
5	2.41/0.46	2.81/0.43	1.98/0.66	1.72/0.48	1.96/0.45	2.66	3.06
6	1.01/0.50	1.25/0.49	0.83/0.71	0.71/0.49	0.86/0.49	1.13	1.39
7	1.23/0.89	2.31/0.82	1.66/1.09	0.90/0.90	1.62/0.84	1.65	2.95



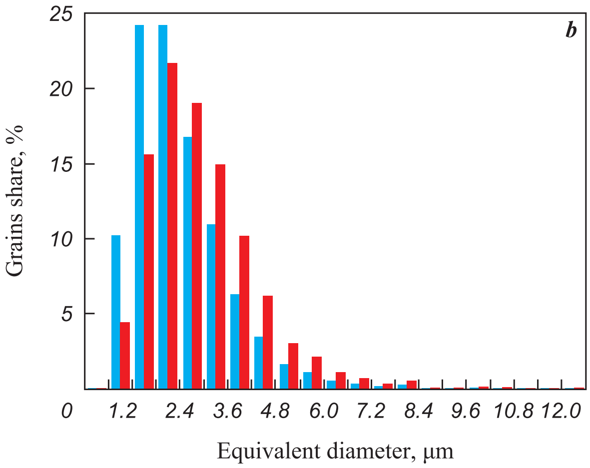


Fig. 1. Demonstration of the shadow effect using the example of a fragment of an image processed in "Fiji" for sample *2* (see Table 1) (***a***) and the equivalent grain diameter distribution for sample *3* with a relatively narrow distribution due to the shadow effect (***b***)

■ – experimental data for all grains; ■ – experimental data for the grains intersected by the lines

Рис. 1. Демонстрация теневого эффекта на примере фрагмента одного из обработанных в «Fiji» снимков для образца *2* (см. табл. 1) (***a***) и распределение зерен по величине эквивалентного диаметра для образца *3* с относительно узким распределением за счет теневого эффекта (***b***)

■ – экспериментальные данные для всех зерен; ■ – экспериментальные данные для зерен на секущих

tion does not provide for the shadow effect, the trend line was drawn through the point (0; 1). As can be seen from Fig. 2, the experimental data are well described by the equation

$$d^{a}/d^{l} = 1 - 0.08c_v - 0.60c_v^2 + 0.13c_v^3, \qquad (1)$$

or

$$d^{a}/d^{l} = 1 - S, \qquad (2)$$

where the shadow function

$$S = 0.08c_v + 0.60c_v^2 - 0.13c_v^3. \qquad (3)$$

It is intuitively assumed that, with a sufficiently large number of grains intersected by the lines, the mean size of such random chords (L) will be equal to the value of the mean chord for all grains (d^{a}_{ch}). The modeling procedure in [14–16] was based on this assumption, and it is valid in the absence of the shadow effect. However, due to the shadow effect, the size L should coincide with the mean chord value for the grains intersected by the line d^{l}_{ch}. Indeed, the ratio for all hardmetals was $L/d^{l}_{ch} = 1.00 \pm 0.03$. Accordingly, taking into account (2), we obtain

$$L = \frac{d^{a}_{ch}}{1 - S}. \qquad (4)$$

When determining the dependence of d^{a}_{eq}/L on the shadow function S, the ratio between the mean values of d^{a}_{eq} and d^{a}_{ch} should be defined. The value

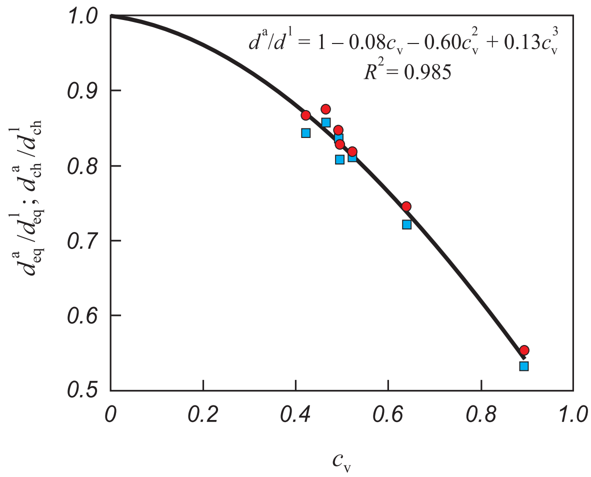


Fig. 2. Plots of d^a_{eq}/d^l_{eq} and d^a_{ch}/d^l_{ch} as function of c_v

Solid line – calculation based on the equation (1)
■ – experimental data for d^a_{eq}/d^l_{eq}
● – experimental data for d^a_{ch}/d^l_{ch}

Рис. 2. Зависимости $d^в_{экв}/d^л_{экв}$ и $d^в_х/d^л_х$ от c_v

Сплошная линия – расчет на основании уравнения (1)
■ – экспериментальные данные для $d^в_{экв}/d^л_{экв}$
● – экспериментальные данные для $d^в_х/d^л_х$

of the ratio $d^{a,l}_{eq}/d^{a,l}_{ch}$ does not depend on the type of alloy grain size distribution and is determined by the grain shape only. Hereinafter, this ratio will be called the shape coefficient K_s. For round grains, the diameter/mean chord ratio is $4/\pi \approx 1.27$. Taking into account averaging over orientations, for rectangular grains, the shape coefficient $K_s \approx 1.36$, for trapezoidal ones – $K_s \approx 1.39$, for triangular ones – $K_s = 1.60 \div 1.70$ depending on the triangle angles. For all the hardmetals under study, no matter what grains were taken for averaging (all grains or the ones intersected by the line), the ratio $d^{a,l}_{eq}/d^{a,l}_{ch} = 1.41 \pm 0.03$ was obtained. Therefore, taking into account the variety of cross-sectional grain shapes of WC–Co hardmetals, the value $K_s \approx 1.4$ seems quite realistic. Hereinafter, we will assume that

$$d^{a,l}_{eq} = 1.4 d^{a,l}_{ch}. \quad (5)$$

Based on (4) and (5), we get the following function:

Table 2. **Measured dimensional parameters (μm/rel. units) for narrow intervals d^l_{eq} for sample *3***

Таблица 2. **Измеренные размерные параметры (мкм/отн.ед.) для узких интервалов $d^л_{экв}$ для образца *3***

d^a_{eq}/c_v	d^l_{ch}/c_v	L/c_v
1.5/0.04	1.05/0.21	1.03/0.44
2.5/0.02	1.78/0.19	1.77/0.45
3.1/0.02	2.21/0.15	2.16/0.43

$$d^a_{eq}/L = 1.4(1 - S). \quad (6)$$

To expand the analyzed range of changes in the width of grain size distributions in the micrographs of sample *3*, we selected the grains with individual sizes d^l_{eq} in three narrow intervals: 1.5 ± 0.06, 2.5 ± 0.06 and 3.1 ± 0.06 μm (analogue of the δ function) and measured the corresponding values of d^l_{ch} and L. Table 2 presents the results obtained for these narrow intervals.

Fig. 3 shows the experimental values of d^a_{eq}/L and $1.4d^a_{ch}/d^l_{ch}$ for the hardmetals under study depending on the width of the WC grain size distribution, as well as the calculated curve according to the equation (6). As can be seen from Fig. 3, the equation (6) presents a comprehensive description of the experimental data.

The relationship between the mean sizes d_J and d_{eq} can be derived from the equation $\sigma^2 = \sum \frac{(d_{eq,i} - d_{eq})^2}{n}$, where σ is the standard deviation for d_{eq}. Since $\sum \frac{d^2_{eq,i}}{n} = d^2_J$, we obtain $\sigma^2 = d^2_J - d^2_{eq}$, or

$$d^{a,l}_J = d^{a,l}_{eq}\sqrt{1 + c_v^2}. \quad (7)$$

In this case, (6) and (7) yield the following equation:

$$d^a_J/L = 1.4(1 - S)\sqrt{1 + c_v^2}. \quad (8)$$

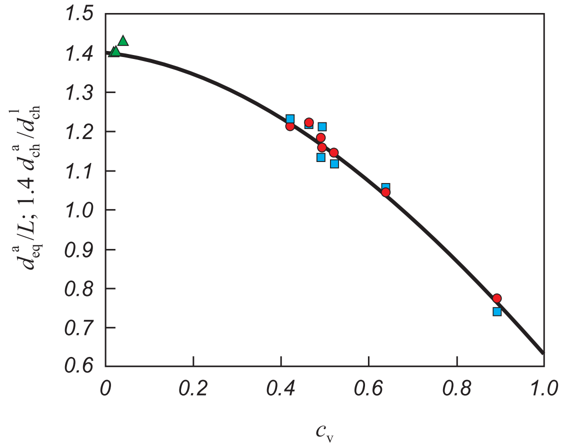


Fig. 3. Plots of d^a_{eq}/L and $1.4d^a_{ch}/d^l_{ch}$ as function of c_v

Solid line – calculation based on the equation (6)
■ – experimental data for d^a_{eq}/L
● – experimental data for $1.4d^a_{ch}/d^l_{ch}$
▲ – experimental data for d^l_{eq}/d^l_{ch} (narrow ranges)

Рис. 3. Зависимости $d^в_{экв}/L$ и $1{,}4d^в_х/d^л_х$ от c_v

Сплошная линия – расчет на основании уравнения (6)
■ – экспериментальные данные для $d^в_{экв}/L$
● – экспериментальные данные для $1{,}4d^в_х/d^л_х$
▲ – экспериментальные данные для $d^л_{экв}/d^л_х$ (узкие диапазоны)

For hardmetals with a different predominant grain shape, equation (8) can be written in a more general form:

$$d_{\mathrm{J}}^{\mathrm{a}}/L = K_{\mathrm{s}}(1-S)\sqrt{1+c_{\mathrm{v}}^2}. \quad (9)$$

Fig. 4 shows the experimental values $d_{\mathrm{J}}^{\mathrm{a}}/L$ and $1.4\sqrt{1+c_{\mathrm{v}}^2}\, d_{\mathrm{ch}}^{\mathrm{a}}/d_{\mathrm{ch}}^{\mathrm{l}}$ for the hardmetals under study, as function of the width of the WC grain size distribution, as well as the calculated curve according to the equation (8).

As can be seen from Fig. 4, the equation (8) presents a satisfactory description of the experiment.

In [14–16], the computer modeling did not take the shadow effect into account and it was assumed that $L \approx d_{\mathrm{ch}}^{\mathrm{a}}$. In this case, the equation (8) becomes

$$\left(d_{\mathrm{J}}^{\mathrm{a}}/L\right)_{\mathrm{model}} = 1.4\sqrt{1+c_{\mathrm{v}}^2}. \quad (10)$$

Fig. 4 also shows the observed dependence of the $d_{\mathrm{J}}^{\mathrm{a}}/d_{\mathrm{ch}}^{\mathrm{a}}$ ratio on c_{v}. It can be seen that the experimental values of $d_{\mathrm{J}}^{\mathrm{a}}/d_{\mathrm{ch}}^{\mathrm{a}}$ and $d_{\mathrm{J}}^{\mathrm{l}}/d_{\mathrm{ch}}^{\mathrm{l}}$ are close to the calculation results [15; 16].

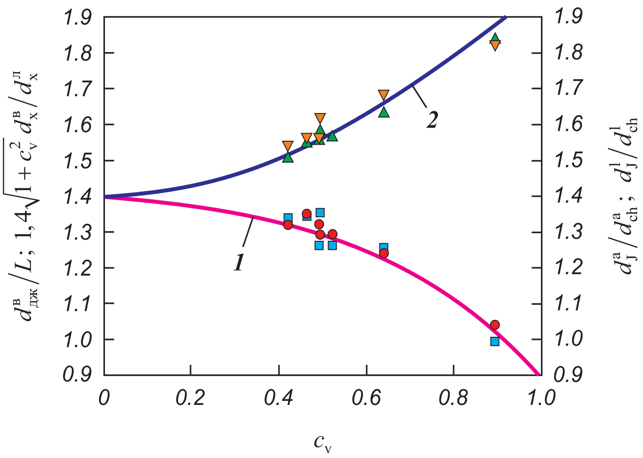


Fig. 4. Plots of $d_{\mathrm{J}}^{\mathrm{a}}/L$, $1.4\sqrt{1+c_{\mathrm{v}}^2}\, d_{\mathrm{ch}}^{\mathrm{a}}/d_{\mathrm{ch}}^{\mathrm{l}}$, $d_{\mathrm{J}}^{\mathrm{a,l}}/d_{\mathrm{ch}}^{\mathrm{a,l}}$ as function of c_{v}

1 – calculation based on the equation (8)

2 – calculation based on the equation (10)

■ – experimental data for $d_{\mathrm{J}}^{\mathrm{a}}/L$

● – experimental data for $1.4\sqrt{1+c_{\mathrm{v}}^2}\, d_{\mathrm{ch}}^{\mathrm{a}}/d_{\mathrm{ch}}^{\mathrm{l}}$

▲ – experimental data for $d_{\mathrm{J}}^{\mathrm{a}}/d_{\mathrm{ch}}^{\mathrm{a}}$

▼ – experimental data for $d_{\mathrm{J}}^{\mathrm{l}}/d_{\mathrm{ch}}^{\mathrm{l}}$

Рис. 4. Зависимости $d_{\text{дж}}^{\text{в}}/L$, $1,4\sqrt{1+c_{\mathrm{v}}^2}\, d_{\text{х}}^{\text{в}}/d_{\text{х}}^{\text{л}}$, $d_{\text{дж}}^{\text{в, л}}/d_{\text{х}}^{\text{в, л}}$ от c_{v}

1 – расчет на основании уравнения (8)

2 – расчет на основании уравнения (10)

■ – экспериментальные данные для $d_{\text{дж}}^{\text{в}}/L$

● – экспериментальные данные для $1,4\sqrt{1+c_{\mathrm{v}}^2}\, d_{\text{х}}^{\text{в}}/d_{\text{х}}^{\text{л}}$

▲ – экспериментальные данные для $d_{\text{дж}}^{\text{в}}/d_{\text{х}}^{\text{в}}$

▼ – экспериментальные данные для $d_{\text{дж}}^{\text{л}}/d_{\text{х}}^{\text{л}}$

In addition to the mean grain size, an important parameter of the microstructure is the WC grain size distribution. As can be seen from Table 1, the variation coefficients of the alloy grain distributions by the equivalent diameters and by mean chords are quite close (difference within 2–3 %). The grain distribution by the mean chord is slightly wider due to the grain shape. In addition, the grain distribution by the mean chords $d_{\mathrm{ch}}^{\mathrm{a}}$ was compared to that by the value of $d_{\mathrm{eq}}^{\mathrm{a}}/1.4$ (normalization by the factor of 1.4 enables to superimpose the distributions on each other). Superposition showed that they coincide within the margin of error. Thus, the alloy grain size distribution by the values of $d_{\mathrm{eq}}^{\mathrm{a}}$ and $d_{\mathrm{ch}}^{\mathrm{a}}$ can equally characterize the alloy grain composition.

As noted above, the size distribution of random chords in the linear intercept method is often used as a characteristic of the alloy grain composition. However, it is necessary to take into account that, firstly, the distribution of random chords in the linear intercept method applies only to grains intersected by the lines, the distribution of which differs from that of all alloy grains due to the shadow effect. Secondly, even if the grains are of the same size and shape (δ-function), the random chords lengths are distributed in a certain way, the width of this distribution is not zero and is determined by the grain shape. For example, if the material consists of round grains of the same diameter d, the mean grain chords are also equal and, respectively, have a δ distribution. In this case, the size distribution density of random chords will be as follows $f(y) = \dfrac{y}{d}\sqrt{d^2 - y^2}$, $y < d$, where d is the circle diameter [21] with the distribution width of $\sigma = 0.223d$. For grains of other shapes, taking possible orientations into account, the distribution function is more complex [22; 23]. Hereinafter, we will call such distribution functions the grain shape functions. Therefore, from a mathematical point of view, the random chord length distribution under the linear intercept method is a convolution of the size distribution function for the grains intersected by the lines and the grain shape function. In many areas of physics, a similar situation can be observed. When random chords distribution is analyzed, it is most logical to use the distribution of mean chords of the grains intersected by the lines as a function of the grain size distribution, while the distribution of random chords for a narrow interval in the distribution of mean chords of the alloy under study can be taken as a shape function. As an illustration, for sample *3* grains intersected by the line, the distributions of mean chords, random chords, and the computed distribution of random chords obtained

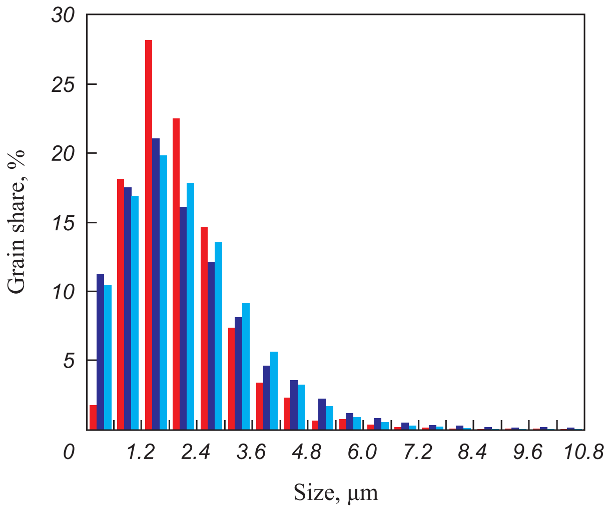


Fig. 5. Grain size distribution by the mean chords (■), random chords (■) and the convolution result (■) for sample *3*

Рис. 5. Распределение зерен по величине средних хорд (■), случайных хорд (■) и результат свертки (■) для образца *3*

by convolution were constructed. The results are shown in Fig. 5.

The comparison of the distribution functions of mean and random chords shows that with equal mean values, the distribution width of random chords is significantly higher (the variation coefficients of the distributions differ 1.4–1.5 times). At the same time, the distribution of random chords, within the margin of error, coincides with the computed distribution obtained by convolution. This means that even for the alloys with relatively narrow grain distributions, when the shadow effect is small, the distribution of random chords will not coincide with the alloy grain size distribution. According to Tikhonov [24], restoring the real function of alloy grain size distribution from the distribution of random chords is a complex task and should be viewed as an incorrect-posed problem.

Discussion of results

The discrepancies between the experimental and computed values of the d_J/L ratio in WC–Co hardmetals revealed in [14–16] indicated a number of problems associated with the linear intercept method. Although this method is widely used, numerous methodological works and a number of international standards are devoted to it, when it comes to WC–Co hardmetals, there is no clear understanding what size is being measured.

The linear intercept method, like the planimetric technique, was originally developed to estimate the mean grain size of polycrystalline metals and alloys, which, as a rule, have rather narrow grain size distributions. Standard WC–Co hardmetals are characterized by a wider WC grain size distribution. This is probably the reason why, with almost similar measurement procedures, the standard for metals and alloys ASTM E112-13 (2021) and the standard for hardmetals ISO 4499 2 (2020) describe different correlations between the linear intercept and planimetric methods.

The ASTM standard indicates the ratio $L = \sqrt{\dfrac{\pi\bar{A}}{4}}$, which is considered accurate for round grains and approximate for equiaxed grains of other shapes, which gives the equation $d_J^a/L \approx 1.273$, or $4/\pi$. As previously noted, this value is equal to the ratio of the circle diameter to its mean chord.

The ISO standard, with reference to [15], indicates the ratio $L = \sqrt{\bar{A}}$, which gives the equation $d_J^a/L \approx 1.128$, or $\sqrt{4/\pi}$. In [15], the experimental values of the ratio d_J^a/L for different alloys varied from 1.10 to 1.40, the mean value being 1.15. Therefore, despite the wide scatter in results, the authors assumed the ratio d_J^a/L to be equal to 1.13 and indicated it in the standard. We believe that the spread in the d_J^a/L value obtained in [15], in addition to the measuring inaccuracy, is attributed to the fact that the alloys probably had different width of the WC grain size distribution, and simple averaging of the d_J^a/L measurement results for different alloys resulted in an error.

The dependence of the mean WC grain size determined by the linear intercept method on the width of the WC grain size distribution revealed in this work and explained by the shadow function S enables to eliminate this error. In addition, the opportunity is afforded, within a single approach, to harmonize the results of the linear intercept and planimetric methods in ASTM E112-13 (2021) and ISO 4499 2 (2020) standards. The ratio d_J^a/L depending on the grain shape and the width of the grain size distribution is given by the equation (9). For round and equiaxed grains, the shape coefficient K_s is approximately equal to 1.27, and for relatively narrow distributions ($c_v < 0.3$) the result from (9) corresponds to ASTM. For hardmetals, due to the variety of shapes of WC grains, the coefficient K_s is approximately equal to 1.4, and the width of the WC grain distribution can vary over a fairly wide range. Therefore, the value of the ratio d_J^a/L, according to (9), can vary from 1.0 to 1.4 depending on c_v. This spread of d_J^a/L values is fully consistent with the experimental results [15]. It confirms that simple averaging of d_J^a/L values for alloys with different c_v values is a mistake and so is the introduction of this mean value (1.128) into ISO 4499 2(2020) standard. Expressing dependence of d_J^a/L on c_v using the equation (8), c_v varying from 0 to 1.0, we obtain an integral mean value of 1.244.

Thus, when the linear intercept method is used to determine the mean WC grain size in hardmetals, the mean size value obtained is even more arbitrary than the researchers, including the authors, earlier assumed. Therefore, this value must be used with caution when establishing a relationship between WC grain size and the physical properties of alloys. It is also important to keep in mind that the distribution of random chords by length in the linear intercept method is not the same as the alloy grain size distribution.

Conclusions

1. For a number of WC–Co hardmetals, having compared the dimensional characteristics of WC grains measured for all grains and for the ones intersected by the line, we proved that the condition for drawing these lines in the linear intercept method (ISO 4499 2) results in shadowing of finer grains by course ones and distortion of the WC grain size distribution (shadow effect).

2. It has been established that the shadow effect grows with the increasing variation coefficient (c_v) of the WC grain size distribution. The relationship between the mean sizes of all grains and the grains intersected by the line can be described using the shadow function S

$$d^a/d^l = 1 - S,$$

where $S = 0.08c_v + 0.60c_v^2 - 0.13c_v^3$.

3. For the hardmetals under study, the relationship between the mean equivalent diameter and the mean chord of WC grains was obtained by measurement:

$$d_{eq} \approx 1.4d_{ch}.$$

4. It is demonstrated that the relationship between the mean equivalent diameter, the Jefferies diameter and the mean grain size in the linear intercept method is not a constant value, it depends on the size of the shadow effect:

$$d_{eq}^a = 1.4L(1 - S),$$

$$d_J^a = 1.4L(1 - S)\sqrt{1 + c_v^2}.$$

5. Without taking the alloy grain size distribution into account, the linear intercept method can only give a conditional estimate of the mean size and these limitations of the method should be kept in mind.

6. The length distribution of random chords in the linear intercept method is not a characteristic of the WC grain size distribution.

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Information about the Authors

Vadim A. Pesin – Leading Expert at the testing laboratory No. 1, "Virial" Ltd.
ORCID: 0000-0002-1250-173X
E-mail: PesinVA@virial.ru

Mariya V. Vasilyeva – Engineer at the testing laboratory No. 1, "Virial" Ltd.
ORCID: 0000-0002-4632-8816
E-mail: VasilyevaMV@virial.ru

Andrey S. Osmakov – Cand. Sci. (Eng.), Head of the testing laboratory No. 1, "Virial" Ltd.
ORCID: 0000-0002-8712-908X
E-mail: OsmakovAS@virial.ru

Сведения об авторах

Вадим Абрамович Песин – вед. специалист испытательной лаборатории № 1, ООО «Вириал»
ORCID: 0000-0002-1250-173X
E-mail: PesinVA@virial.ru

Мария Владимировна Васильева – инженер испытательной лаборатории № 1, ООО «Вириал»
ORCID: 0000-0002-4632-8816
E-mail: VasilyevaMV@virial.ru

Андрей Сергеевич Осмаков – к.т.н., начальник испытательной лаборатории № 1, ООО «Вириал»
ORCID: 0000-0002-8712-908X
E-mail: OsmakovAS@virial.ru

Contribution of the Authors

V. A. Pesin – formulating the main concept of the article, determining the purpose of the work, making calculations, writing the article.
M. V. Vasilyeva – making measurements and calculations, writing the article.
A. S. Osmakov – defining the tasks of the work, amending the manuscript, participating in the analysis and discussion of the results.

Вклад авторов

В. А. Песин – формирование основной концепции статьи, определение цели работы, проведение расчетов, написание статьи.
М. В. Васильева – проведение измерений и расчетов, написание статьи.
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